

# The non dissipative damping of the Rabi oscillations as a “which-path” information

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Rabi oscillations may be viewed as an interference phenomenon due to a coherent superposition of different quantum paths, like in the Young's two-slit experiment. The inclusion of the atomic external variables causes a non dissipative damping of the Rabi oscillations. More generally, the atomic translational dynamics induces damping in the correlation functions which describe non classical behaviors of the field and internal atomic variables, leading to the separability of these two subsystems. We discuss on the possibility of interpreting this intrinsic decoherence as a “which-way” information effect and we apply to this case a quantitative analysis of the complementarity relation as introduced by Englert [Phys. Rev. Lett. **77**, 2154 (1996)].

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Young's double-slit experiment may be considered as an emblematic example to introduce the wave-particle duality, which, to quote Feynman [1], is the basic mystery of Quantum Mechanics (QM). This experiment is frequently discussed in the introductory part of QM textbooks [2] to explain how the interference pattern and the which-way information on the particle trajectory are mutually exclusive behaviors in QM. Moreover, this wave-particle dualism belongs to the enlarged contest of complementarity [3, 4], which has its root in the superposition principle. Coherent superposition of two states or, more generally, of two quantum paths leads to interference effects which may manifest itself as probability oscillations of some “populations”. For example, when a two-level atom interacts with the field of an optical cavity, the atomic level populations undergo to the well known Rabi oscillations. But, what is the interference effect underlying this phenomenon? In other words, which paths do interfere?

When the atomic external degrees of freedom are included in the model, the internal dynamics of the atom correlates with the translational variables. This information transfer towards the atomic external variables is at the origin of an intrinsic decoherence effect, that is, it causes a non dissipative damping of the Rabi oscillations [5, 6]. Non dissipative damping of Rabi oscillations has been observed experimentally [7, 8] and it has also been analyzed by Bonifacio *et. al.* who consider the evolution time as random variable [9]. In this letter we will show that a damping also affects all the correlations functions which are involved in the Bell inequality [10] and in the separability criterion [11, 12] for the field and atomic internal variables. Can this decoherence be interpreted as a “which-way” information effect? In this Letter we will try to throw light on these questions.

Let us consider a two-level atom interacting with the field of an optical cavity. In the rotating wave approximation (RWA) the atom-field interaction is described by the Hamiltonian  $\hbar\epsilon \sin k\hat{x}(\hat{a}^\dagger \hat{S}_- + \hat{a} \hat{S}_+)$ , where the usual spin 1/2 operators refer to the internal dynamics of the two-level atom, while  $\hat{a}$  and  $\hat{a}^\dagger$  are the annihilation and creation operators for the photons of k-mode of the cavity standing wave and  $\epsilon$  is the atom-field coupling constant. The atom enters the cavity moving prevalently along a direction orthogonal to the x-cavity axis, and we assume that the atomic velocity along this direction is large enough to treat classically this component of the motion. This interaction is at the heart of the optical Stern-Gerlach effect [13], and it takes a form very similar to the magnetic case introducing a new set of “spin” operators

$$(\hat{\mu}_x, \hat{\mu}_y, \hat{\mu}_z) = \left( \frac{\hat{a}^\dagger \hat{S}_- + \hat{a} \hat{S}_+}{2\sqrt{\hat{N}}}, i \frac{\hat{a}^\dagger \hat{S}_- - \hat{a} \hat{S}_+}{2\sqrt{\hat{N}}}, \hat{S}_z \right), \quad (1)$$

which satisfy in fact the algebra of a spin 1/2,  $[\hat{\mu}_x, \hat{\mu}_y] = i\hat{\mu}_z$  et cycl., where  $\hat{N} = \hat{a}^\dagger \hat{a} + \hat{S}_z + \frac{1}{2}$ . We assume that the transverse spatial distribution of the incoming atom is given by a packet of width  $\Delta x_0$  narrow with respect to the wavelength  $\lambda$  of the resonant mode, and centered near a nodal point of the cavity function. In these conditions the sinusoidal mode function of the cavity standing-wave can be approximated by the linear term [14]. In the resonance conditions the optical Stern-Gerlach Hamiltonian reads

$$\hat{H} = \frac{\hat{p}^2}{2m} + \hbar\omega\hat{N} + \hbar\hat{\Omega}_x\hat{\mu}_x, \quad (2)$$

where  $\hat{p}$  is the conjugate momentum of the position observable  $\hat{x}$ ,  $m$  is the mass of the particle and  $\hat{\Omega}_x = 2\epsilon k\sqrt{\hat{N}}\hat{x}$ . The kinetic term of this Hamiltonian accounts for the atomic translational degree of freedom along the x-direction.

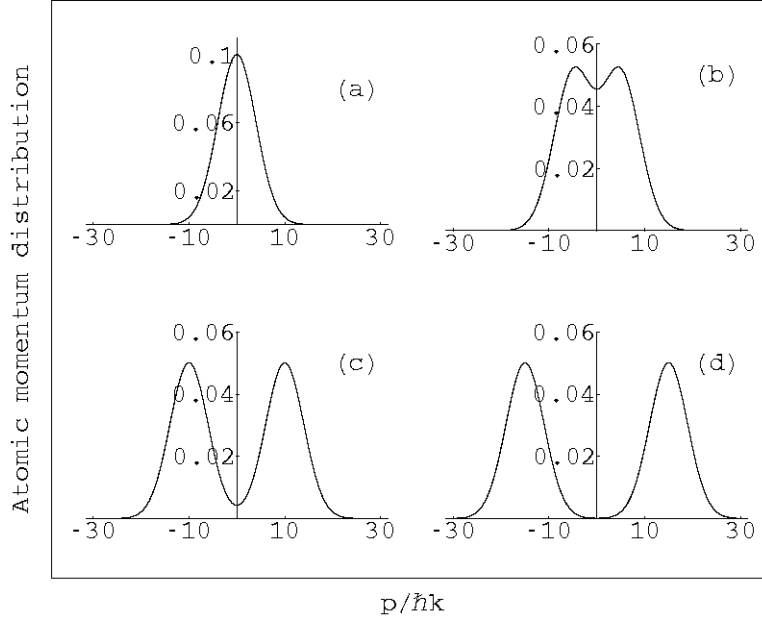


FIG. 1: Momentum distribution (Eq.(7)) for a two-level atom entering the cavity in the state  $|\varphi(0)\rangle|e\rangle$  and interacting with the vacuum state  $|0\rangle$  of the cavity field;  $\varphi(p,0)$  is a Gaussian packet of minimum uncertainty, with zero mean value of  $\hat{p}$  and  $\Delta p_0/\hbar k = 25/2\pi$ , corresponding to  $\Delta x_0/\lambda = 1/50$ . Curve (a) describes the initial distribution at time  $t = 0$ . Curves (b), (c) and (d) refer to the interaction times  $\epsilon T = 5$ ,  $\epsilon T = 10$  and  $\epsilon T = 15$ , respectively, where  $\epsilon = 10^8 \text{ sec}^{-1}$ . The values of the other parameters are  $m = 10^{-26} \text{ kg}$ ,  $\lambda = 10^{-5} \text{ meter}$ .

Consider the one excitation initial configuration

$$|\psi(0)\rangle = |e, 0\rangle |\varphi(0)\rangle = \frac{1}{\sqrt{2}} (|\chi^+\rangle + |\chi^-\rangle) |\varphi(0)\rangle, \quad (3)$$

where  $|\varphi(0)\rangle$  is the initial ket of the translational dynamics along the cavity axis, and we have expanded the ket  $|e, 0\rangle \equiv |e\rangle|0\rangle$  in terms of the dressed states  $|\chi^\pm\rangle = \frac{1}{\sqrt{2}}(|e, 0\rangle \pm |g, 1\rangle)$  which are eigenstates of the excitation number operator,  $\hat{N}|\chi^\pm\rangle = |\chi^\pm\rangle$ , and of the interaction energy,  $\hat{\mu}_x|\chi^\pm\rangle = \pm\frac{1}{2}|\chi^\pm\rangle$ . As usually,  $|e\rangle$  and  $|g\rangle$  indicate the upper and the lower states of the internal atomic dynamics, while  $|n\rangle$  is a field number state. Using these relations and the evolution operator  $\hat{U}(t, 0) = e^{-i\hat{H}t/\hbar}$  we finally obtain the state of the entire system at time  $t \leq T$ , ( $T$  is the cavity flight time)

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} (|\phi^+(t)\rangle |\chi^+\rangle + |\phi^-(t)\rangle |\chi^-\rangle), \quad (4)$$

where

$$|\phi^\pm(t)\rangle = \exp(-i\frac{t}{2m\hbar}\hat{p}^2) \exp(\mp\frac{i}{2\hbar}at^2\hat{p}) \exp(\mp\frac{i}{\hbar}mat\hat{x}) |\varphi(0)\rangle \quad (5)$$

account for the splitting of the translational state into two parts, which go away in opposite sides, with a mean acceleration  $a = \hbar k \epsilon / m$ . In fact, in the  $p$ -representation we have

$$\phi^\pm(p, t) = \langle p | \phi^\pm(t) \rangle = \varphi(p \pm mat, 0) \exp\left[-i\frac{pt}{2\hbar}\left(\frac{p}{m} \mp at\right)\right]. \quad (6)$$

Using the orthonormality of the dressed states  $|\chi^\pm\rangle$ , the atomic momentum distribution is

$$|\psi(p, t)|^2 = \frac{1}{2} \left[ |\phi^+(p, t)|^2 + |\phi^-(p, t)|^2 \right]. \quad (7)$$

This probability density, shown in Fig.1 for some values of the interaction time, displays the well known optical Stern-

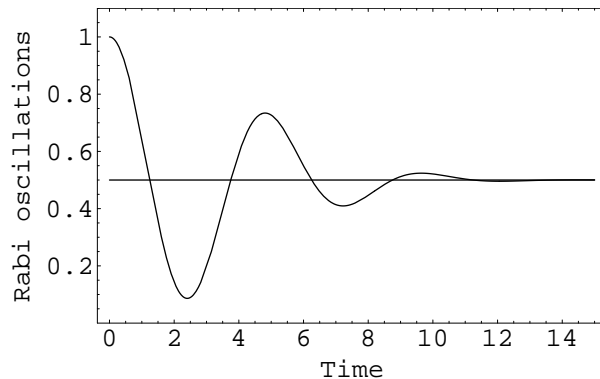


FIG. 2: Time evolution of the atomic population (Eq. (8)). The time is in  $\epsilon^{-1}$  units and  $x_0 = \frac{\lambda}{10}$ . The values of the other parameters are as in Fig.1. A similar damping affects the correlation functions of Eqs.(12) and (13), which state the conditions for the violation of the Bell's inequality and for the separability, respectively.

Gerlach effect. The mechanical action of the light causes the splitting of the atomic packet into two peaks which become distinguishable for interaction time sufficiently large (a few period of the Rabi frequency). This behaviour owns some interesting and quite surprising features. The exchange of momentum (through the photons exchange) between the atom and the cavity walls cannot be a full random process since, in this case, we would expect a single peak, centered in the initial mean value of  $\hat{p}$ . On the contrary, both the Fig.1 and the Eqs.(4) and (5) suggest that this exchange can follow, in a single experiment, only one of two mutually exclusive quantum paths. The momentum exchange behaves as only one cavity side or the other should be involved with the same probability (because of the particular initial state), and the Rabi oscillations do emerge as a consequence of a coherent superposition of these two quantum paths. According to this scenario, it has been recently shown [6] that the one-sided atomic deflection accompanies with the so-called atomic coherent trapping [15], in which the atomic internal population is trapped into its initial value. In this extreme case the particular initial configuration will cause the disappearing of one of the two quantum paths, and the Rabi oscillations are absent since the beginning. Certainly, the which-way information on these two possibilities cannot be acquired by looking at the walls of the cavity, which are absent from the model (2) (this is equivalent to assume an infinite mass for the walls). However, we can obtain an indirect which-path information by looking at the momentum distribution of the deflected atom. For interaction times sufficiently large, the two peaks of the atomic momentum are distinguishable (see curves (c) and (d) of Fig.1), and, in the single experiment, one may be acquainted with the quantum path the system has followed. At the same time, the interference effects go to zero, as the damping of the Rabi oscillations shows (see Fig.2).

The correlations between the internal and the translational atomic dynamics as displayed in Eq.(4) lead to a drastic change in the time behavior of the internal variables with respect to the usual Jaynes-Cummings model, in which the atomic external variables are disregarded. Let  $P(e, t)$  indicate the probability of finding the atom in the excited state at time  $t$ ,

$$P(e, t) = \frac{1}{2} [1 + \text{Re} (\langle \phi^-(t) | \phi^+(t) \rangle)], \quad (8)$$

where the relation  $\hat{\mu}_z |\chi^\pm\rangle = \frac{1}{2} |\chi^\mp\rangle$  has been utilized. For example, if the initial translational state is given by a Gaussian distribution of minimum uncertainty  $\Delta x_0 \Delta p_0 = \hbar/2$ , centered in  $x_0$  and with zero mean velocity along the cavity axis, for the scalar product of Eq.(8) we have

$$\langle \phi^-(t) | \phi^+(t) \rangle = e^{-i\Omega t} \exp \left\{ -\frac{[x^+(t) - x^-(t)]^2}{8\Delta x_0^2} - \frac{[p^+(t) - p^-(t)]^2}{8\Delta p_0^2} \right\}, \quad (9)$$

where  $x^\pm(t) = x_0 \mp a t^2/2$ ,  $p^\pm(t) = \mp m a t$  and  $\Omega = 2 m a x_0/\hbar$ . As Eq.(9) shows, the scalar product between the two translational components  $|\phi^\pm(t)\rangle$  depends on the distance in the phase space of the same components. When the distance is sufficiently large the scalar product goes to zero, the two paths become distinguishable and their interference effects vanish. We wish to outline that this non dissipative damping of the Rabi oscillations shown in Fig.2, may be considered as an intrinsic decoherence effect: It originates inside the optical Stern-Gerlach model of Eq.(2), without claiming any sort of reservoir action. It is to notice that the modulus of the scalar product (9) essentially measures the *visibility*  $V$  (of the Rabi oscillations) as given by Englert [16]. We recall that, in our case, the translational dynamics plays the role of the *detector*, i.e., with the notation of Ref.[16],  $\rho_D^{(i)} = |\varphi(0)\rangle \langle \varphi(0)|$ ,

$\rho_D^{(+)} = |\phi^+\rangle\langle\phi^+|$  and  $\rho_D^{(-)} = |\phi^-\rangle\langle\phi^-|$ . Similarly it is possible to obtain the complementary quantity

$$D = \left(1 - |\langle\phi^+|\phi^-\rangle|^2\right)^{\frac{1}{2}}. \quad (10)$$

The *distinguishability*  $D$  assumes a very simple form because the initial state of the detector is a pure state. As pointed by Englert, “the ways cannot be distinguished at all if  $D = 0$ ” (when  $V=1$ ) “and they can be held apart completely if  $D = 1$ ” (when  $V=0$ ). Due to the purity of the initial state, the equality

$$D^2 + V^2 = 1 - |\langle\phi^+|\phi^-\rangle|^2 + |\langle\phi^+|\phi^-\rangle|^2 = 1$$

is satisfied at any time. In this contest the appearance of the damping of the Rabi oscillations can be related to a *visibility* degradation. On the other hand, since the quantum paths become distinguishable and a which-way information is accessible, the quantum nature of the atom-field correlations is vanishing. Consider the reduced density operator describing the field and the atomic internal dynamics. Giving up the information about the external variables by tracing the density operator of the state Eq.(4) over the translational variables we get

$$\rho = \frac{1}{2} \left[ |\chi^+\rangle\langle\chi^+| + |\chi^-\rangle\langle\chi^-| + (\langle\phi^-(t)|\phi^+(t)\rangle |\chi^+\rangle\langle\chi^-| + h.c.) \right]. \quad (11)$$

We now investigate the nature of the atom-field correlations in terms of the Bell’s inequality. Because of its simplicity, we consider the Horodecki family formulation [17], which is equivalent to the standard Clauser, Horne, Shimony, Holt, (CHSH) formulation [18], when a bipartite system of spin 1/2 is involved, as in our case. The test reads: *A density matrix  $\rho$  describing a system composed by two spin 1/2 subsystems violates some Bell’s inequality in the CHSH formulation if and only if the relation  $M(\rho) > 1$  is satisfied.* The quantity  $M(\rho)$  can be defined as follows. Consider the  $3 \times 3$  matrix  $T_\rho$  with coefficients  $t_{n,m} = \text{tr}(\rho \sigma_n \otimes \sigma_m)$ , where  $\sigma_n$  are the standard Pauli matrices. Diagonalizing the symmetric matrix  $U_\rho = T_\rho^T \cdot T_\rho$  ( $T_\rho^T$  is the transpose of  $T_\rho$ ), and denoting two greater eigenvalues of  $U_\rho$  by  $\lambda_1$  and  $\lambda_2$ , then  $M(\rho) = \lambda_1 + \lambda_2$ . In our case we find

$$M(\rho) = 1 + [\text{Im}(\langle\phi^-(t)|\phi^+(t)\rangle)]^2. \quad (12)$$

Looking at the Eqs. (3) and (9) it is evident that our system, which is initially non correlated, becomes, as the atom-field interaction time increases, non-locally correlated. Quite interestingly, at the same time the atomic external dynamics, which makes accessible the which-way information, starts to damp this non-locality. Since the violation of the Bell’s inequality is just a sufficient condition for non-locality, it may be useful to consider the separability [11, 12] of the density matrix because it is a sufficient condition for both the locality and the classical correlation. After a few periods of Rabi oscillations the damping factor involved in the scalar product (9), determines the separability of the system in the sense that it is possible to set it in the form  $\rho = \sum_{r=1}^{\infty} p_r W_r^{(1)} \otimes W_r^{(2)}$ , where  $W_r^{(1)}$  and  $W_r^{(2)}$  are states corresponding to the single subsystems and  $p_r$  are probabilities ( $\sum_{r=1}^{\infty} p_r = 1$ ). To test this possibility in a more quantitative way we consider the eigenvalues of the partial transpose [11]  $\sigma_\rho$  of the matrix  $\rho$ . A necessary and sufficient condition for separability reads [11, 19]:  *$\rho$  is separable if and only if all the eigenvalues of  $\sigma_\rho$  are non-negative.* In our case the test writes

$$|\text{Im}(\langle\phi^-(t)|\phi^+(t)\rangle)| \leq 0. \quad (13)$$

Because of the correlations with the atomic external variables the classicality of the correlations between the field and the atomic internal variables is recovered, as explicitly shown by the diagonal form  $\rho = \frac{1}{2}(|e\rangle\langle e| \otimes |0\rangle\langle 0| + |g\rangle\langle g| \otimes |1\rangle\langle 1|)$  that the density matrix (11) assumes after a few Rabi periods (when  $\langle\phi^-(t)|\phi^+(t)\rangle \rightarrow 0$ ). It is intriguing the fact that the which-way information can be accessible just when the quantum nature of the atomic translational dynamics is taken into account.

In conclusion, the interaction between the cavity field and the internal atomic variables can pursue two mutually exclusive quantum paths. Due to the correlation with the atomic external variables, these two paths cause the well known optical Stern-Gerlach effect, that is, the atomic translational packet splits into two parts. When these are sufficiently separate, a which way information becomes accessible, and all the correlations functions involved in the Bell’s inequality, in the separability criterion, and in the Rabi oscillations go exponentially to zero, showing a propensity towards a classical behavior. In a forthcoming paper we will use the same model to prove how the atomic translational effects make more difficult the violation of Bell’s inequality for massive particles.

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